APPENDIX E

ACTUARIAL ANALYSIS EXAMPLE

1.0 INTRODUCTION

The E-6 aircraft is a Boeing 707 airframe modified for the Navy for strategic communications. In this role, the aircraft is fitted with a dual trailing wire antenna (DTWA) system. The DTWA is a hydro-mechanical driven antenna and reel system. The DTWA consists of two separate antenna reeling assemblies, the short trailing wire assembly (STWA) and the long trailing wire assembly (LTWA). Each assembly was analyzed separately. The LTWA is presented in this example.

2.0 PROBLEM IDENTIFICATION

When the Navy procured the E-6 aircraft, the LTWA system was removed from Navy EC-130Q aircraft, overhauled, and installed in the E-6. While on the EC-130Q, the LTWA was overhauled every 4 years, coincident with the aircraft's SDLM cycle. The E-6 was expected to have a 5 to 7 year SDLM cycle so the question was raised, "Can the LTWA overhaul be extended to 5 to 7 years?" To answer that question the actuarial analysis presented in this example was performed.

It is noted in passing that the one asking the question is presupposing the need for an overhaul. This, at first may seem to be a valid presupposition since the system is currently being overhauled. The better question, however, is "Does the LTWA need to be overhauled and if so when?" This example will answer the latter question.

To answer the question at hand it is necessary to determine if the LTWA exhibits a wear out age. This is evaluated in accordance with the RCM philosophy adopted by the Navy. For the system to exhibit a wear out age it must be possible to identify a point in time where the conditional probability of failure curve shows a rapid increase. The actuarial analysis process used to determine this is documented in MIL-STD-2173 and NAVAIR 00-25-403. FIGURE 1 provides a graphical representation of this process.
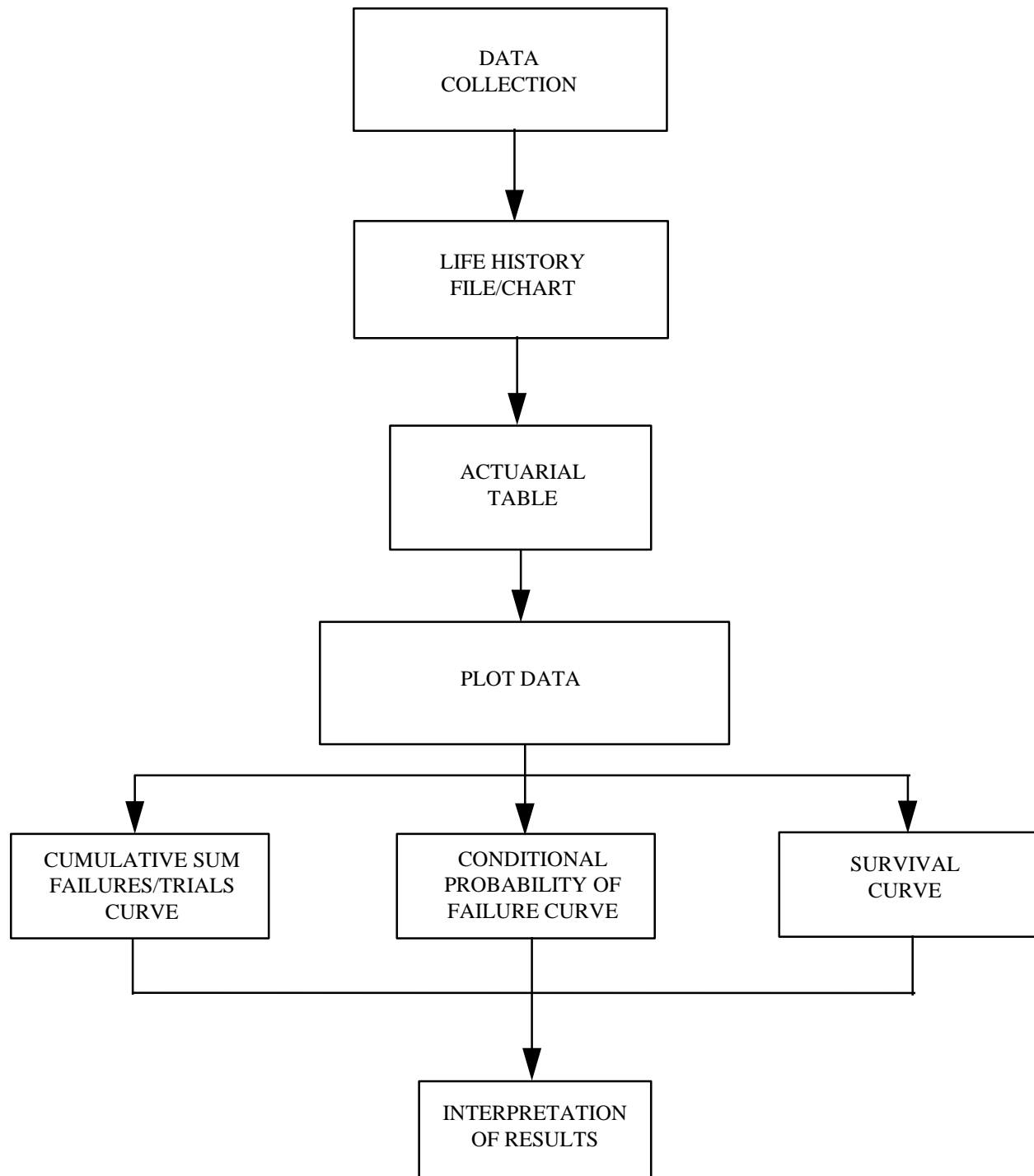
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| CUMULATIVE SUM FAILURES/TRIALS CURVE | CONDITIONAL PROBABILITY OF FAILURE CURVE | SURVIVAL CURVE |

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                    ┌─────────────────────┐
                    │  INTERPRETATION     │
                    │   OF RESULTS        │
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FIGURE 1. Actuarial Analysis

3.0 METHODOLOGY

Actuarial analysis is the processing of life data to determine, from an appropriate sample, the effect of aging on the probability of failure. The usual objective of the analysis is the determination of the effectiveness of a scheduled rework or discard task. Actuarial analysis can be performed in two basically different ways; either from life test data in which a selected group of units all start at zero age and are operated until all have failed, or from a larger group of in-service life data segments bounded by two calendar dates. The second method was used for the analysis presented in this example.

3.1 ACTUARIAL ANALYSIS

The actuarial analysis begins with the collection of failure data for the system or a similar system. For this analysis, failure data for the LTWA system was available since the system had been operating on the EC-130Q for a number of years. A ten year period (1 January 1980 through 31 December 1989) was chosen for analysis. These dates were chosen for 2 primary reasons.

(1) the data was available and relatively consistent. Data consistency prior to 1980 was more questionable.

(2) The LTWA is only expected to operate for 10 years before being replaced with a newer system. The failure data was extracted from the 3-M system.

After collecting the data the maintenance actions were categorized into failures and non-failures. The 3-M data is reported with a number of codes that identify when the defect was found, the type of defect, and the repair action. Also an ECA software package was available that defines the appropriate combinations of these codes for a verified failure. Therefore, the data could be categorized as those maintenance actions that were verified as failures and those that were not.

The next step is to group the data into time blocks. For this analysis the data was originally grouped into 3 month blocks. The failures and non-failures documented against each aircraft were plotted. Figure 2 and Figure 3 provide the Life History Chart data for the LTWA analysis. The X's signify maintenance actions that were verified as failures and the O's signify those that were not. It is noted that in some quarters a single aircraft may have multiple maintenance actions with or without failures. This is a complication that will be considered later. It is also noted that the entire assemblies were not routinely removed from an aircraft until they were overhauled. When an assembly was removed for overhaul, a new or newly overhauled assembly was put

in its place. This made tracking by aircraft a good measure of
the effectiveness of overhaul and made data collection and
analysis easier. Each aircraft would receive 2 new or newly
overhauled assemblies during the 10 year period of study.
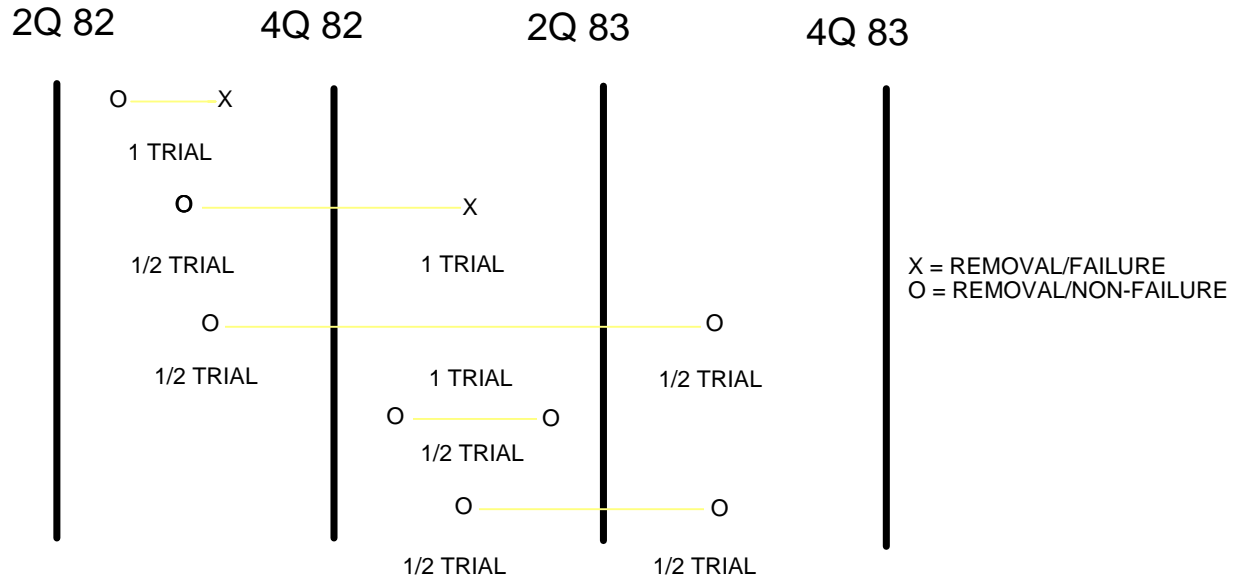
ACTUARIAL ANALYSIS
TRIAL DEFINITIONS

FIGURE 2. Actuarial Analysis Trial Definitions

3.2 CALCULATIONS

The Actuarial Table contained in Table 1 provides the results of the calculations described in this section.

AGE INTERVALS	# FAILURES	WHOLE TRIALS	TOTAL TRIALS	FAILURES/ TRIALS	CUMULATIVE SUM FAILURES/TRIALS	FARED CURVE	MID CELL	SURVIVAL PLOT
2Q 80	3	12	19.5	0.1538462	0.153846	-0.431		1.431
4Q 80	10	21	28	0.3571429	0.5109889	0.103	0.534	0.666846
2Q 81	9	20	22	0.4090909	0.9200798	0.637	0.534	0.3107502
4Q 81	6	20	22.5	0.2666667	1.1867464	1.172	0.535	0.1444989
2Q 82	9	20	23	0.3913043	1.5780508	1.706	0.534	0.0673365
4Q 82	9	19	23.5	0.3829787	1.9610295	2.241	0.535	0.0313115
2Q 83	15	24	30	0.5	2.4610295	2.775	0.534	0.0145911
4Q 83	15	27	30.5	0.4918033	2.9528328	3.31	0.535	0.0067849
2Q 84	10	17	25.5	0.3921569	3.3449896	3.844	0.534	0.0031618
4Q 84	18	26	29.5	0.6101695	3.9551591	4.378	0.534	0.0014734
2Q 85	28	33	39	0.7179487	4.6731079	4.913	0.535	0.0006851
4Q 85	38	41	50	0.76	5.4331079	5.447	0.534	0.0003193
2Q 86	21	28	34	0.6176471	6.0507549	5.982	0.535	0.0001485
4Q 86	10	17	21	0.4761905	6.5269454	6.516	0.534	0
2Q 87	19	28	32.5	0.5846154	7.1115608	7.051	0.535	0
4Q 87	20	29	32.5	0.6153846	7.7269454	7.585	0.534	0
2Q 88	20	27	31	0.6451613	8.3721067	8.119	0.534	0
4Q 88	13	24	32	0.40625	8.7783567	8.654	0.535	0
2Q 89	22	29	34.5	0.6376812	9.4160378	9.188	0.534	0
4Q 89	10	26	27.5	0.3636364	9.7796742	9.723	0.535	0
2Q 90	23	35	41.5	0.5542169	10.333891	10.257	0.534	0
4Q 90	10	22	25	0.4	10.733891	10.791	0.534	0

Table 1. Actuarial Table

The actuarial analysis process now requires that the number of "trials" be calculated. FIGURE 2 is provided to help visualize trials and half trials. If the system operates through the time period without a failure or non-failure maintenance action a full trial is counted. Entering or leaving during the time period without failure counts as a half trial. Finally, a failure during the time period is a full trial. This is where the multiple failures and non-failures maintenance actions within a time period became a challenge. How should these be counted? Should each failure be counted as a trial or should all of the failures be counted as one trial? Similarly, should each non-failure be counted as a half trial or should all non-failures be counted as one half trial? To test the effect of each combination, all possible combinations were analyzed. It turned out that the conclusions of the analysis were the same for all combinations. The rest of this example will use each failure as a full trial and each non-failure as a half trial. Finally, to simplify the analysis the number of trials were calculated for 6 month time blocks.

The next step is to count the total number of failures. Again all of the failures in the time block were counted. After obtaining the total number of failures and the total trials the probability of failing in each time block is calculated by dividing the total

number of failures by the total trials. Finally, the time block
probability of failures are cumulated and plotted as shown on
FIGURE 3.

It is now necessary to fair a curve through the plotted points to
smooth the line. Faring the curve can be manually drawn or
calculated with curve fitting equations. Since the plot appeared
to be linear, linear regression analysis techniques were used.
The resulting fared curve equation is Y = 0.534X - 0.431, with a
Correlation Coefficient of 0.997. The smoothed data points are
now added to Table 1.

3.2.1 WEAROUT AGE

The question of how one knows when an overhaul task is applicable
is now addressed. An overhaul task is applicable when the system
or component exhibits a wear out age. In statistical terms, the
wear out age is determined by a point at which the conditional
probability of failure curve shows a rapid increase. The data
has been prepared to develop the conditional probability of
failure curve.

The conditional probability of failure curve is developed from
the smoothed data points. The conditional probability of failure
for each time block is calculated by subtracting it's smooth data
point from the previous time blocks. Mathematically:

$$MC_{n+1} = FC_{n+1} - FC_n$$

The conditional probability of failure is interpreted as the
probability that the system or component will fail during the
time block given that it survived until that time. The
conditional probability of failure curve for the LTWA is shown on
FIGURE 3. Finally, a survivability curve is calculated using the
following equation:

$$S_{n+1} = (1 - MC_{n+1}) * S_n$$

The survivability curve is used to evaluate the effectiveness of
the overhaul interval. The overhaul interval is only effective
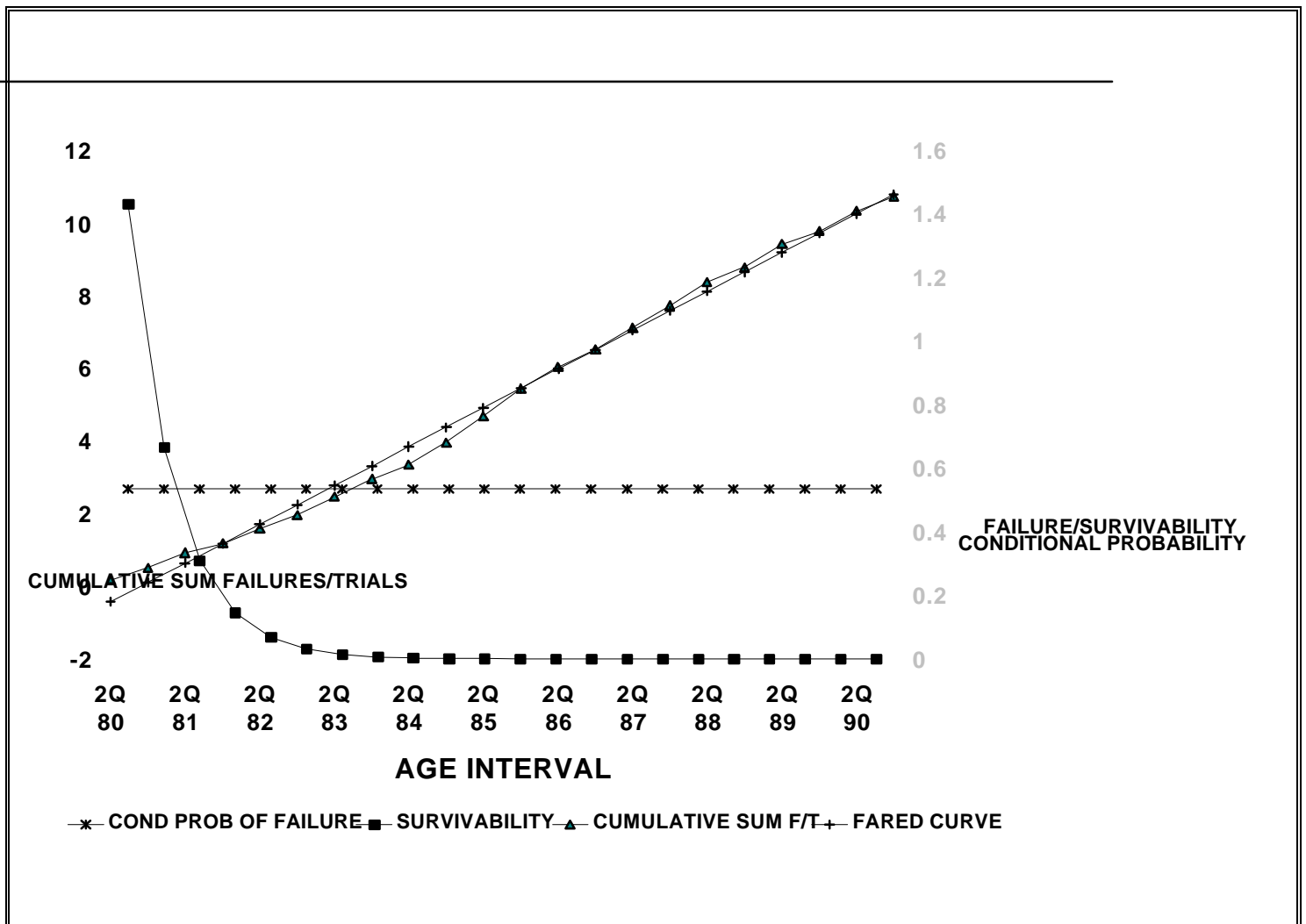if an adequate percentage of the population survives to the wear
out age.

FIGURE 3. Actuarial Graph

4.0 INTERPRETATION OF THE DATA

The heart of the actuarial analysis is in the conditional probability of failure curve. In this case the conditional probability of failure curve is a horizontal line. This is interpreted to say that the probability of failure of the system the day after overhaul is exactly the same as the probability of failure the day before overhaul. Since there is no point at which the probability of failure curve shows a rapid increase, a wear out age is not found.

5.0 CONCLUSION

Recall that the original question was "Can the overhaul be extended from 4 to 5 or 7 years?" The question was modified to "Is an overhaul applicable?" Since a wearout age is not exhibited (at least in 10 years) an overhaul is not applicable.

It is noted that a wear out age might be found at a time greater than 10 years but the expectation is that the horizontal line will continue for many years. In any case the equipment will be replaced before a wear out age is reached.

6.0 **RESULT**

Since an overhaul was not found to be applicable for either the LTWA or STWA, it was recommended that a complete RCM analysis be performed on the DTWA to determine the appropriate preventive maintenance tasks to maintain the system. The RCM analysis was performed and implemented. Upon implementation of the RCM analysis, it was calculated that replacing the overhaul with other appropriate field level preventive maintenance saved approximately $35-40 Million over 7 years.

REFERENCES

STANDARDS

    MILITARY

MIL-STD-2173 (AS)   Reliability-Centered Maintenance
                        Requirements For Naval Aircraft, Weapons
                        Systems and Support Equipment

    OTHER GOVERNMENT DOCUMENTS, DRAWINGS, AND PUBLICATIONS

NAVAIR 00-25-403   Management Manual, Guidelines for the Naval
                        Aviation Age Exploration Program

DEFINITIONS

The terms and acronyms listed in this example are defined as follows:

1. DTWA: Dual Trailing Wire Antenna

2. LTWA: Long Trailing Wire Antenna

3. STWA: Short Trailing Wire Antenna

4. SDLM: Standard Depot Level Maintenance

5. RCM: Reliability Centered Maintenance

6. 3-M: Maintenance Material Management System

7. ECA: Equipment Condition Analysis

8. Cell: Segment of a Life History Chart

9. Trial: An attempt of the system/unit to cross a life segment boundary

10. Whole Trial

    A. A system/unit enters a cell at the lower boundary and makes a successful traverse through the whole cell and continues into the next cell.

    B. A system/unit enters a cell at the lower boundary and fails within the cell.

    C. A system/unit starts within a cell and fails within that cell.

11. Half Trial

    A. A system/unit enters a cell at the lower boundary and is removed from the data set without failure while in that cell.

    B. A system/unit starts within a cell and either makes a successful traverse or is removed from the data set without failure while in that cell.

12. Verified Failure (ECA): An action for which:

    A. The action taken code is:

1). Repair or replace of items, such as attaching units, seals, gaskets, packing, tubing, hose and fittings, things that are not integral parts of the system/unit.

or

2). Repair, which includes, cleaning, disassembly, inspection, reassembly, lubrication, and replacement of integral parts.

or

3). Corrosion treatment, which includes cleaning, treatment, priming, and painting of corroded items that require no other repair.

and

B.   The malfunction code is unconditional, meaning a fault (failure or not) occurred requiring removal of the system/unit.

13. Cumulative Sum Failures/Trials Curve: A graph of the cumulative sum of total failures/trials as a function of age depicting the relationship between failures and age.

14. Conditional Probability of Failure Curve: A graph of the probability that an item will fail during a particular age interval, given that it survives to enter that interval.

15. Survival Curve: A graph of the probability of survival of an item as a function of age, derived by actuarial analysis of its service history. The area under the curve can be used to measure the average realized age of the item.